



LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

B.Sc.DEGREE EXAMINATION –PHYSICS

THIRD SEMESTER – APRIL 2019

PH 3506– MATHEMATICAL PHYSICS

Date: 24-04-2019
Time: 01:00-04:00

Dept. No.

Max. : 100 Marks

PART-A

Answer ALL Questions

(10×2=20 marks)

1. Find the square root of $-4-3i$.
2. Evaluate $\oint \frac{3z^2+7z+1}{z+1} dz$, where C is the circle with $|z| = \frac{1}{2}$
3. Find the directional derivative of $w(x, y, z) = x^2yz + 4xz^2$ at $(1, -2, 1)$ in the direction of $2\hat{i} - \hat{j} - 2\hat{k}$.
4. Define double and triple integrals with one example each.
5. Define half range Fourier series. Give example.
6. State Dirichlet's conditions for a Fourier series.
7. prove that the matrix $(1/3) \begin{bmatrix} 1 & 1+i \\ 1-i & -1 \end{bmatrix}$ is unitary
8. What do you mean by orthogonal matrix? Give one example.
9. State Simpson's 1/3 rule.
10. Write down the normal equations for fitting a straight line $y=a+bx$ by the method of least squares.

PART-B

Answer ANY FOUR Questions

(4×7.5=30 marks)

11. Prove that $u = x^2 - y^2 - 2xy - 2x + 3y$ is harmonic. Find a function v such that $f(z) = u+iv$ is analytic.
Also express $f(z)$ in terms of z .
12. A fluid motion is given by $\vec{v} = (y+z)\hat{i} + (z+x)\hat{j} + (x+y)\hat{k}$. Show that the motion is irrotational and hence find the velocity potential.
13. Expand as a Fourier series, the output of a half wave rectifier.
14. Define rank of a matrix. Find the rank of a matrix $\begin{bmatrix} 1 & 3 & 4 & 2 \\ 2 & -1 & 3 & 2 \\ 3 & -5 & 2 & 2 \\ 6 & -3 & 8 & 6 \end{bmatrix}$ by reducing it to normal form.
15. Solve the following system of equations by using Gauss-Seidal method correct to 3 decimal places.
 $8x-3y+2z = 20$
 $4x+11y-z = 33$
 $6x+3y+12z = 35$

16. (i). Derive Newton-Gregory forward interpolation formula for equal intervals (4 marks).

(ii). Find the values of y at x=21 from the following data

x : 20 23 26 29
y: 0.3420 0.3907 0.4384 0.4848 (3.5 marks)

PART-C

Answer ANY FOUR Questions

(4×12.5=50 marks)

17. (i) State and prove Cauchy's integral theorem (6 marks).

(ii) Use Cauchy's integral formula to evaluate $\int_C \frac{z}{z^2 - 3z + 2} dz$, Where C is the circle

$$|Z - 2| = 1/2.$$

(6.5 marks)

18. (a) State and prove Stoke's theorem (7marks).

(b) Use Gauss divergence theorem to evaluate $\iint_S \vec{A} \cdot d\vec{s}$ where $\vec{A} = x^3 \hat{i} + y^3 \hat{j} + z^3 \hat{k}$ and S is the surface of the sphere $x^2 + y^2 + z^2 = a^2$ **(5.5 marks).**

19. (i) If $A = xz^3 \hat{i} - 2x^2yz \hat{j} + xyz^4 \hat{k}$. Find $\nabla \times A$ at the point (1, -1, 1). **(5 marks).**

(ii) Find the inverse of a matrix A by applying elementary transformation

$$A = \begin{bmatrix} 0 & 2 & 1 & 3 \\ 1 & 1 & -1 & -2 \\ 1 & 2 & 0 & 1 \\ -1 & 1 & 2 & 6 \end{bmatrix}$$

(7.5marks).

20. (i) Find the Fourier sine transform of $1/x$. **(7.5marks)**

(ii) Obtain a Fourier expansion of $f(x) = x^3$, for $-\pi < x < \pi$ (5marks).

21. Find the Eigen values and Eigen vectors of a given matrix A by writing its characteristic equation ,

$$A = \begin{bmatrix} 3 & 1 & 4 \\ 0 & 2 & 6 \\ 0 & 0 & 5 \end{bmatrix}$$

22. (i) Compute y at x=0.25 by modified Euler's method. Given $y' = 2xy, y(0) = 1$ **(6.5 marks)**

(ii) Evaluate the integral $I = \int_4^{5.2} \log_e x \, dx$ using Trapezoidal rule and Simpson's 1/3 rule.

(6 marks)
